Macerata 30 ottobre 2015 – classe 4M – COMPITO DI MATEMATICA

SOLUZIONE FILA 1

1) Utilizzando le formule di duplicazione si ha:

$$sen\frac{10}{7}\alpha = sen\left(2\cdot\frac{5}{7}\alpha\right) = 2sen\frac{5}{7}\alpha\, cos\frac{5}{7}\alpha$$

$$cos\frac{10}{7}\alpha = cos\left(2\cdot\frac{5}{7}\alpha\right) = 2\,cos^2\frac{5}{7}\alpha - 1 = 2\frac{1}{4} - 1 = -\frac{1}{2}$$

Per risolvere il problema serve calcolare $sen\frac{5}{7}\alpha$ in funzione di $cos\frac{5}{7}\alpha$. Si ha: $sen\frac{5}{7}\alpha = -\sqrt{1-cos^2\frac{5}{7}\alpha} = -\sqrt{1-\frac{1}{4}} = -\frac{\sqrt{3}}{2}$ in quanto l'angolo di $\frac{5}{7}\alpha$ sta nel quarto quadrante e quindi il seno è negativo. Si ha:

$$sen\frac{10}{7}\alpha = sen\left(2\cdot\frac{5}{7}\alpha\right) = 2sen\frac{5}{7}\alpha\, cos\frac{5}{7}\alpha = 2\left(-\frac{\sqrt{3}}{2}\right)\frac{1}{2} = -\frac{\sqrt{3}}{2}$$

2) Utilizzando le formule di bisezione si ha

$$sen\frac{\pi}{12} = sen\left(\frac{\pi/6}{2}\right) = \sqrt{\frac{1-cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2-\sqrt{3}}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$cos\frac{\pi}{12} = cos\left(\frac{\pi/6}{2}\right) = \sqrt{\frac{1+cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2+\sqrt{3}}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$tg\frac{\pi}{12} = tg\left(\frac{\pi/6}{2}\right) = \sqrt{\frac{1-cos\frac{\pi}{6}}{1+cos\frac{\pi}{6}}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}} = \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}} = 2-\sqrt{3}$$

3)

$$\sqrt{3}\cdot\frac{cos\,3\alpha - cos\,\alpha}{sen\left(\frac{\pi}{6}+\alpha\right) - sen\left(\frac{\pi}{6}-\alpha\right)} = -2sen2\alpha$$

Utilizzo le formule di prostaferesi nel termine di sinistra

$$\sqrt{3}\cdot\frac{-2sen\frac{3\alpha+\alpha}{2}\,sen\frac{3\alpha-\alpha}{2}}{2\,cos\frac{\frac{\pi}{6}+\alpha+\frac{\pi}{6}-\alpha}{2}\,sen\frac{\frac{\pi}{6}+\alpha-\frac{\pi}{6}+\alpha}{2}} = -2sen2\alpha$$

$$\sqrt{3}\cdot\frac{-\cancel{2}\,sen2\alpha\,\cancel{sen\alpha}}{\cancel{2}\,cos\frac{\pi}{6}\,\cancel{sen\alpha}} = -2sen2\alpha$$

$$-\cancel{\sqrt{3}}\cdot\frac{sen2\alpha}{\frac{\cancel{\sqrt{3}}}{2}} = -2sen2\alpha \qquad\qquad -2sen2\alpha = -2sen2\alpha$$

4)

$$\frac{sen2\alpha \cdot sen4\alpha}{cos\,\alpha} = cos\,3\alpha - cos\,5\alpha$$

Utilizzo le formule di duplicazione per il seno a sinistra e prostaferesi a destra

$$\frac{2sen\alpha \cancel{cos\,\alpha} \cdot sen4\alpha}{\cancel{cos\,\alpha}} = -2sen\frac{3\alpha+5\alpha}{2}\,sen\frac{3\alpha-5\alpha}{2}$$

$$2sen\alpha \cdot sen4\alpha = -2sen4\alpha sen(-\alpha)$$

$$2sen\alpha \cdot sen4\alpha = 2sen4\alpha \cdot sen\alpha$$

5) Calcola il valore della seguente espressione

$$2\,cos\left(\frac{3}{8}\pi+\alpha\right)cos\left(\frac{3}{8}\pi-\alpha\right) = cos\,2\alpha - \frac{\sqrt{2}}{2}$$

Utilizzo le formule di Werner a sinistra

$$2\frac{1}{2}\left[cos\left(\frac{3}{8}\pi+\alpha+\frac{3}{8}\pi-\alpha\right)+cos\left(\frac{3}{8}\pi+\alpha-\frac{3}{8}\pi+\alpha\right)\right] = cos\,2\alpha-\frac{\sqrt{2}}{2}$$

$$cos\frac{3}{4}\pi + cos\,2\alpha = cos\,2\alpha - \frac{\sqrt{2}}{2} \qquad -\frac{\sqrt{2}}{2} + cos\,2\alpha = cos\,2\alpha - \frac{\sqrt{2}}{2}$$

6)

$$cotg\,\frac{\gamma}{2} = \frac{sen\alpha + sen\beta}{cos\,\beta + cos\,\alpha}$$ Utilizzo le formule di prostaferesi a destra

$$cotg\,\frac{\gamma}{2} = \frac{\cancel{2}\,sen\frac{\alpha+\beta}{2}\,\cancel{cos\frac{\alpha-\beta}{2}}}{\cancel{2}\,cos\frac{\alpha+\beta}{2}\,\cancel{cos\frac{\alpha-\beta}{2}}}$$

$$cotg\,\frac{\gamma}{2} = \frac{sen\frac{\alpha+\beta}{2}}{cos\frac{\alpha+\beta}{2}} = tg\,\frac{\alpha+\beta}{2}$$ essendo $\alpha+\beta = \pi-\gamma$

$$cotg\,\frac{\gamma}{2} = tg\,\frac{\pi-\gamma}{2} \qquad cotg\,\frac{\gamma}{2} = tg\left(\frac{\pi}{2}-\frac{\gamma}{2}\right)$$

SOLUZIONE FILA 2

1) Utilizzando le formule di duplicazione si ha:

$$sen\frac{6}{5}\alpha = sen\left(2\cdot\frac{3}{5}\alpha\right) = 2sen\frac{3}{5}\alpha\, cos\frac{3}{5}\alpha$$

$$cos\frac{6}{5}\alpha = cos\left(2\cdot\frac{3}{5}\alpha\right) = 2cos^2\frac{3}{5}\alpha - 1 = 2\frac{1}{4} - 1 = -\frac{1}{2}$$

Per risolvere il problema serve calcolare $sen\frac{3}{5}\alpha$ in funzione di $cos\frac{3}{5}\alpha$. Si ha: $sen\frac{3}{5}\alpha = -\sqrt{1-cos^2\frac{3}{5}\alpha} = -\sqrt{1-\frac{1}{4}} = -\frac{\sqrt{3}}{2}$ in quanto l'angolo di $\frac{3}{5}\alpha$ sta nel terzo quadrante e quindi il seno è negativo. Si ha:

$$sen\frac{6}{5}\alpha = 2sen\frac{3}{5}\alpha\, cos\frac{3}{5}\alpha = 2\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$$

2) Calcola

$$sen\frac{\pi}{8} = sen\left(\frac{\pi/4}{2}\right) = \sqrt{\frac{1-cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

$$cos\frac{\pi}{8} = cos\left(\frac{\pi/4}{2}\right) = \sqrt{\frac{1+cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$tg\frac{\pi}{8} = tg\left(\frac{\pi/4}{2}\right) = \sqrt{\frac{1-cos\frac{\pi}{4}}{1+cos\frac{\pi}{4}}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}} = \sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}} = \sqrt{2}-1$$

3)

$$\frac{cos\,3\alpha + cos\,\alpha}{sen\left(\frac{\pi}{6}+\alpha\right) + sen\left(\frac{\pi}{6}-\alpha\right)} = 2cos\,2\alpha$$

Utilizzo le formule di prostaferesi nel termine di sinistra

$$\frac{2cos\frac{3\alpha+\alpha}{2}\,cos\frac{3\alpha-\alpha}{2}}{2sen\frac{\frac{\pi}{6}+\alpha+\frac{\pi}{6}-\alpha}{2}\,cos\frac{\frac{\pi}{6}+\alpha-\frac{\pi}{6}+\alpha}{2}} = 2cos\,2\alpha$$

$$\frac{\cancel{2}\,cos\,2\alpha\,\cancel{cos\,\alpha}}{\cancel{2}\,sen\frac{\pi}{6}\,\cancel{sen\,\alpha}} = 2cos\,2\alpha$$

$$\frac{cos\,2\alpha}{\frac{1}{2}} = 2cos\,2\alpha \qquad 2cos\,2\alpha = 2cos\,2\alpha$$

4)

$$\frac{sen2\alpha \cdot cos\,4\alpha}{cos\,\alpha} = sen5\alpha - sen3\alpha$$

Utilizzo le formule di duplicazione per il seno a sinistra e prostaferesi a destra

$$\frac{2sen\alpha\,\cancel{cos\,\alpha}\cdot cos\,4\alpha}{\cancel{cos\,\alpha}} = 2\,cos\frac{5\alpha+3\alpha}{2}\,sen\frac{5\alpha-3\alpha}{2}$$

$$2sen\alpha \cdot cos\,4\alpha = 2\,cos\,4\alpha \cdot sen\alpha$$

5) Calcola il valore della seguente espressione

$$2sen\left(\frac{5}{12}\pi+\alpha\right)cos\left(\frac{5}{12}\pi-\alpha\right) = sen2\alpha+\frac{1}{2}$$

Utilizzo le formule di Werner a sinistra

$$2\,\frac{1}{2}\left[sen\left(\frac{5}{12}\pi+\alpha+\frac{5}{12}\pi-\alpha\right)+sen\left(\frac{5}{12}\pi+\alpha-\frac{5}{12}\pi+\alpha\right)\right] = sen2\alpha+\frac{1}{2}$$

$$sen\frac{5}{6}\pi+sen2\alpha = sen2\alpha+\frac{1}{2} \qquad \frac{1}{2}+sen2\alpha = sen2\alpha+\frac{1}{2}$$

6)

$$tg\,\frac{\gamma}{2} = \frac{sen\alpha - sen\beta}{cos\,\beta - cos\,\alpha}$$ Utilizzo le formule di prostaferesi a destra

$$tg\,\frac{\gamma}{2} = \frac{\cancel{2}\,cos\frac{\alpha+\beta}{2}\;\cancel{sen\frac{\alpha-\beta}{2}}}{\cancel{-}\,\cancel{2}\,sen\frac{\alpha+\beta}{2}\;\cancel{sen\frac{\beta-\alpha}{2}}}$$

$$tg\,\frac{\gamma}{2} = \frac{cos\frac{\alpha+\beta}{2}}{sen\frac{\alpha+\beta}{2}}$$ essendo $\alpha+\beta=\pi-\gamma$

$$tg\,\frac{\gamma}{2} = \frac{cos\left(\frac{\pi}{2}-\frac{\gamma}{2}\right)}{sen\left(\frac{\pi}{2}-\frac{\gamma}{2}\right)} = \frac{sen\frac{\gamma}{2}}{cos\frac{\gamma}{2}}$$

prostaferesi

$$sen\alpha + sen\beta = 2sen\frac{\alpha+\beta}{2}cos\frac{\alpha-\beta}{2}$$

$$sen\alpha - sen\beta = 2\,cos\frac{\alpha+\beta}{2}sen\frac{\alpha-\beta}{2}$$

$$cos\,\alpha + cos\,\beta = 2\,cos\frac{\alpha+\beta}{2}cos\frac{\alpha-\beta}{2}$$

$$cos\,\alpha - cos\,\beta = -2sen\frac{\alpha+\beta}{2}sen\frac{\alpha-\beta}{2}$$

$$tg\alpha \pm tg\beta = \frac{sen(\alpha\pm\beta)}{cos\,\alpha\, cos\,\beta}$$

WERNER

$$sen\alpha\, cos\,\beta = \frac{1}{2}\left[sen(\alpha+\beta) + sen(\alpha-\beta)\right]$$

$$cos\,\alpha\, cos\,\beta = \frac{1}{2}\left[cos(\alpha+\beta) + cos(\alpha-\beta)\right]$$

$$sen\alpha sen\beta = -\frac{1}{2}\left[cos(\alpha+\beta) - cos(\alpha-\beta)\right]$$